Multimode network representation for H- and E-plane uniform bends in rectangular waveguide

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Abstract

In this paper we describe new multimode network representations for both H- and E-plane uniform bends in terms of impedance and admittance multimode coupling matrices, respectively. The key element of the network is the transition from the straight waveguide to the curved waveguide. The relevant multimode equivalent network representation is obtained following a simple procedure that has been used already with success for other types of junctions involving straight waveguides. In the talk, the details of the formulations will be discussed together with comparisons between our simulations and available published data, both measured and theoretical, indicating very good agreement as well as very good computational efficiency.

I - Introduction

Uniform bends in rectangular waveguides, like the ones shown in Fig. 1, are frequently used components in many microwave subsystems for both ground and space applications. Their accurate and efficient full-wave characterization is therefore required for the development of modern CAD tools to analyze and design complex waveguide structures. Many contributions can be found in the technical literature concerning bends (see [1] to [9] for instance) but no publications are actually available (to the authors knowledge) describing multimode equivalent network representations.

In this paper we describe new multimode network representations for both H- and E-plane uniform bends in terms of impedance and admittance multimode coupling matrices, respectively. The key element of the network is the transition from the straight waveguide to the curved waveguide. The relevant multimode equivalent network representation is obtained following a simple procedure that has been used already with success for other types of junctions involving straight waveguides. First the theoretical formulation is discussed in details together with its convergence properties. Next a number of comparisons are presented between our simulations and data already available in the technical literature, both theoretical and measured, indicating that the approach proposed is at the same time accurate and computationally very efficient.

II - Theory

The viewpoint chosen to simulate a waveguide bend consists of the cascading of two discontinuities through a length L of transmission line. The discontinuities are the junctions between straight to curved and curved to straight waveguide regions, while the the length of transmission line represents the uniform curved region (see Fig. 2). As a consequence, the first step toward the development of the equivalent network representation of the structures in Fig. 1, in the form shown in Fig.2, is the computation of an orthonormal set of modes for the curved waveguide section. To this end, the expansions proposed by Lewin [4] have been used in order to avoid the direct use of Bessel functions.

The procedure described in [4] involves the expansion of the transverse electric and magnetic fields as an infinite series of standard rectangular waveguide basis functions $\mathbf{e}_n^{(s)}$, $\mathbf{h}_n^{(s)}$, namely (H-plane bend case)

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$$\mathbf{e}_m^{(c)} = \sum_r d_r^{(m)} \mathbf{e}_r^{(s)} \qquad (1)$$

$$\mathbf{h}_m^{(c)} = \sum_r d_r^{(m)} \mathbf{h}_r^{(s)} \qquad (2)$$

$$\mathbf{e}_n^{(s)} = -\sqrt{\frac{2}{a}} sin(\frac{n\pi}{a}(x+\frac{a}{2}))\mathbf{y}_0 \qquad (3)$$

$$\mathbf{h}_n^{(s)} = \sqrt{\frac{2}{a}} sin(\frac{n\pi}{a}(x+\frac{a}{2}))\mathbf{x_0} \quad (4)$$

where (c) and (s) denote the curved and the straight waveguide regions, respectively. Furthermore, the appropriate orthogonality conditions, called in the reminder the overlapping integrals, are

$$< f,g >^{(c)} = \int_{-a/2}^{a/2} \frac{1}{1+\frac{x}{R}} f(x)g(x)dx \quad (5)$$

$$< f,g >^{(s)} = \int_{-a/2}^{a/2} f(x)g(x)dx \quad (6)$$

The series (1) and (2) are then inserted in the Helmothz equation of the curved region, obtaining an eigenvalue problem solved using a Galerkin procedure so that the propagation constants and the coefficients of the series expansion are finally obtained. A similar procedure was also used successfully by Weisshaar [7].

Once the modes of the curved waveguide section are obtained, we can proceed with the formulation of the network by following a new simple method that has been already used to analyze the junction between arbitrary straight waveguides [10] and [11]. First, we need to define two reference planes denoted as T and T', as shown in Fig. 1. We can then write the mathematical equivalent of the network representation in Fig. 2 (H-plane bend case) in the form

$$V_m^{(\delta)} = \sum_{n=-\infty}^{n=\infty} Z_{m,n}^{(\delta,s)} I_n^{(s)} + \sum_{n=-\infty}^{n=\infty} Z_{m,n}^{(\delta,c)} I_n^{(c)} \quad (7)$$

where $V_m^{(\delta)}$ and $I_m^{(\delta)}$ are the modal voltages and currents, respectively. According to circuit theory, the $Z_{m,n}^{(\delta,\gamma)}$ element is given by the general relation

$$Z_{m,n}^{(\delta,\gamma)} = \frac{V_m^{(\delta)}}{I_n^{(\gamma)}} \mid I_k^{(\xi)} = 0 \ \forall \xi \neq \gamma \, \text{and} \, k \neq n \quad (8)$$

Eq. (8) can be used to actually evaluate the $Z_{m,n}^{(\delta,\gamma)}$ elements, resulting in the following simple expressions:

$$Z_{m,n}^{(s,s)} = -jZ_m^{(s)} cotg(\beta_m^{(s)} l)\delta_{m,n} \quad (9)$$

$$Z_{m,n}^{(s,c)} = -jZ_n^{(s)} cosec(\beta_n^{(s)} l)\frac{1}{I_n^{(s)}} < \mathbf{e}_n^{(s)}, \mathbf{e}_m^{(c)} >^{(c)} \quad (10)$$

$$Z_{m,n}^{(c,c)} = \frac{-j}{I_n^{(c)}} \sum_{k=1}^{K} Z_k^{(s)} cotg(\beta_k^{(s)} l) < \mathbf{e}_k^{(s)}, \mathbf{e}_m^{(c)} >^{(c)} < \mathbf{h}_n^{(c)}, \mathbf{h}_k^{(s)} >^{(s)} \quad (11)$$

where K is the maximum number of modes summed in the series (3) and (4) . The derivation yielding the above expressions will be discussed in details during the talk and is not included here for the sake of space. What is important to note, however, is that the overlapping integrals involved in the process to obtain the elements of the impedance (admittance) matrix are not frequency-dependent, and that only the $Z_{m,n}^{(c,c)}$ elements involve a summation.

The above impedance matrix describes a single straight to curved junctions. More complicated structures can now be easily analyzed by connecting several equivalent network representations through appropiate transmission line lengths. At the end of the cascading process, only one banded linear system has to be inverted, resulting in a very fast code implementation. The process to analyze a E-plane bend involves the same steps, but the equivalent network representation is formulated instead in terms of admittance matrices.

III - Numerical and experimental results

For the sake of space, only results for a H-plane bends are presented here (more examples will be presented in the talk). Fig. 3 shows the convergence of the $Z_{m,n}^{(c,c)}$ element as function of the number of terms to describe each mode. Only 10 expansion terms are enough to obtain good convergence. In Fig. 4 the convergence of the magnitude of the reflection coefficient with the number of modes in the global network is analyzed, showing that typically only 4 or 5 modes are required. In order to validate the computational method presented here, we next compare our results with the theoretical results presented by Weisshaar [7] in Fig. 5. As we can see, a very good agreement is observed. Computation time for a typical analysis (with 5 modes in the network and 10 terms to describe each mode of the curved region) with 50 points in frequency takes 2 seconds on a IBM RISK-6000 workstation. Finally, as a further verification, we present a comparison between measured results (curtesy of Radiacion y Microondas, S.A.) and our simulations in Fig. 6. Also in this case, the agreement is very good thereby further validating the presented model.

IV - Conclusions

A new multimode equivalent network representation for the analysis of uniform H- and E-plane bends in rectangular waveguide has been presented. The junction between straight and curved waveguide regions, which is the key element of the structure, is analyzed in terms of a multimode equivalent network representation involving impedance or admittance coupling matrix for H- and E-plane bends, respectively. The convergence of the network representation has been analyzed as function of several parameters showing very good behaviour. Comparison with theoretical and experimental results fully validate the method presented.

V - Acknowledgment

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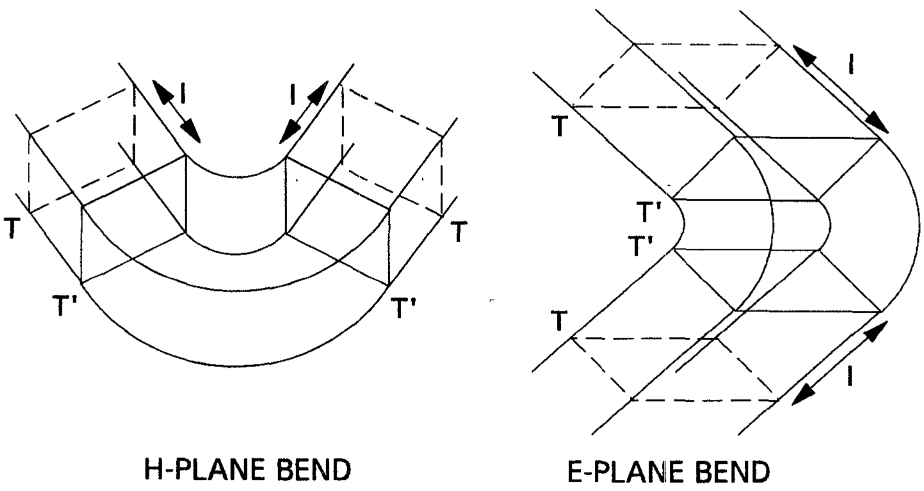

Fig. 1 Uniform H- and E-plane bends in rectangular waveguide analyzed in this paper.

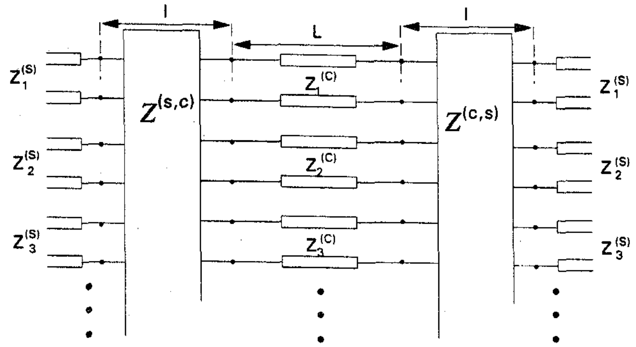

Fig. 2 Multimode equivalent network for H-plane bend. The $Z^{(s,c)}$ ($Z^{(c,s)}$) impedance matrix represents the junction straight and curved (curved and straight) waveguide regions.

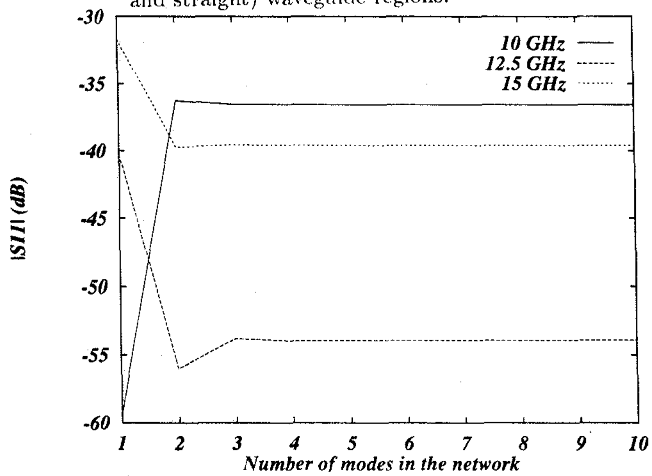

Fig. 4 Convergency of the magnitude of the reflection coefficient versus the number of modes included in the final network. 90° H-plane bend in WR-75 waveguide. Radius=21.6 mm. 10 expansion basis have been used to describe each mode.

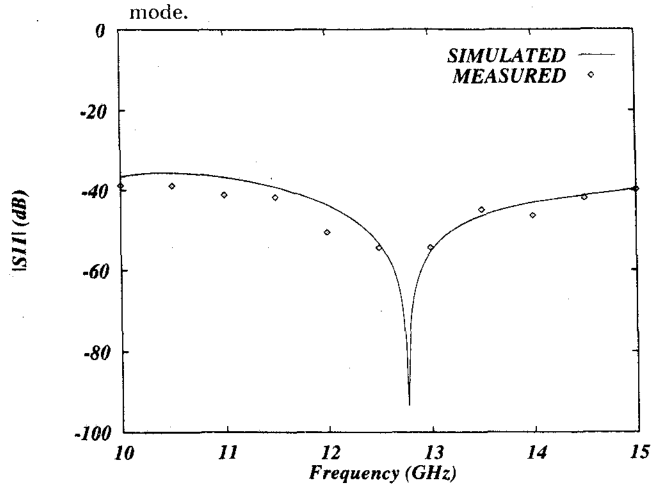

Fig. 6 Comparison of the measured data with our approach. 90° H-plane bend in WR-75 waveguide. Radius=21.6 mm. 5 modes have been included in the network. (a) Magnitude of the reflection coefficient, (b) Phase of the transmission coefficient.

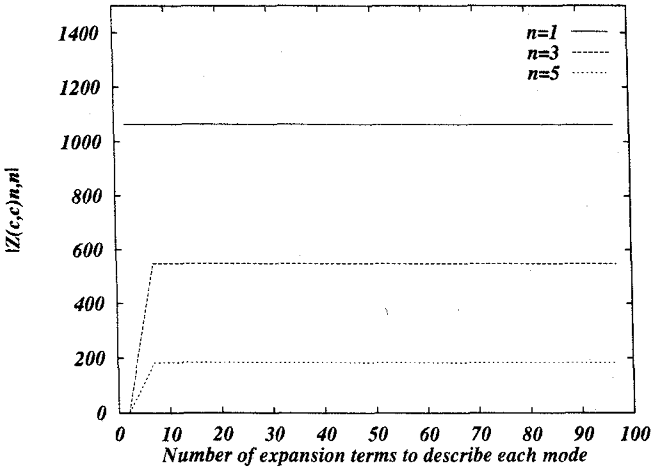

Fig. 3 Convergency of the $Z_{m,n}^{(c,c)}$ element as function of the number of expansion basis used to describe each curved region mode. WR-75 waveguide. Radius=21.6 mm, Frequency=14 GHz.

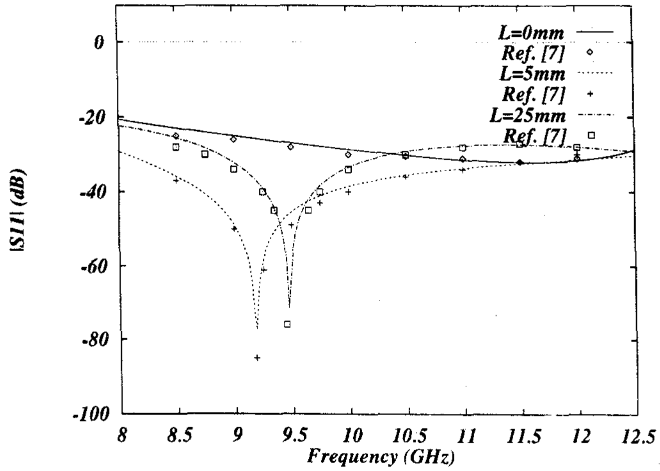

Fig. 5 Comparison of the results presented in [7] with our method. Cascaded 30° H-plane bends through a straight transmission line of length L. WR-90 waveguide. Radius=15.24 mm. U-configuration. 5 modes have been included in the network.